

Wind-Directed Solution Diversity in Abhijit's TSP Algorithm: A Novel Exploration Heuristic via Directional Cost Bias

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Abstract

We report an empirical discovery arising from an interactive visualisation of Abhijit's TSP Algorithm: applying a directional cost bias ('wind') to the distance matrix during route construction produces meaningfully different local optima, with the best wind direction consistently improving on the no-wind result. Across 20 trials at 40 cities, the SPXQ strategy with wind exploration improves on its baseline by an average of 4.93-5.26% and helps in 85-90% of trials. The best result across all 8 wind directions beats the current BEST mode (all 8 strategies, no wind) in 20 of 30 trials. We propose this as a novel solution-diversity mechanism for TSP local search and outline directions for further investigation.

1. Discovery

1.1 A Ten-Day Arc

The companion paper to this note — presenting Abhijit's Algorithm itself — was first published on 15 August 2026. At that point the algorithm's behaviour on a fixed Euclidean plane was believed to be fully characterised: eight construction strategies, a best-improvement refinement loop, and benchmark results against known optima. Ten days later, the results in this paper did not exist even as a hypothesis.

What changed in between was not months of theoretical analysis but a short, iterative cycle of building an interactive visual feature, watching it run, noticing something unexpected, and following that observation wherever it led. Adding a wind/current overlay was intended as a demonstration of the algorithm's generality — proof that it could handle any cost matrix, not just Euclidean distance. Watching the seeding phase visibly change shape under different wind directions was not part of that plan; it was noticed only because the visualisation made the construction process observable step by step, not just as a final number.

Eric Temple Bell, introducing *Men of Mathematics* (1937), argued that the first rough attempt at a hard problem by a serious investigator can teach more than generations of subsequent refinement of an already-polished method — that early, direct engagement with a problem has its own kind of illumination, distinct from and sometimes exceeding what patient later polishing provides. The wind-exploration finding, the bias-sensitivity discovery, and the case study in Section 4.5 all arose the same way: not from planning to find them, but from staying close enough to the running algorithm to notice when it did something that didn't fit the existing picture, and taking that seriously rather than dismissing it as noise.

During interactive experimentation with a wind/current overlay added to the TSP visualiser at tsp.uncledroid.app, it was observed that changing the wind direction while using the SPX (farthest neighbour) seeding strategy produced visibly different optimal tours. This prompted systematic investigation of whether directional cost bias could serve as a solution-diversity mechanism for the algorithm's construction phase.

The mechanism is as follows. A global directional bias modifies the cost matrix such that travel in the wind direction is cheaper and travel against it more expensive. For a CLOSED tour, the net effect of this bias always cancels mathematically — the round trip covers equal upwind and downwind distance. However, during the OPEN chain construction phase (SPN, SPX, SSF, SPEN and their variants), the algorithm makes greedy choices based on the biased costs. Different wind directions bias these choices differently, producing different starting chains, which the refinement loop then optimises to different local optima.

This is equivalent to placing the problem on a tilted plane: the same cities, the same geometry, but viewed from a different directional perspective. The algorithm explores different regions of the solution space without any randomisation or parameter tuning — purely through systematic rotation of the cost landscape.

2. Experimental Design

2.1 Wind Cost Model

For travel from point A to point B with wind direction θ and bias strength $\beta \in [0, 0.75]$:

```
dx = B.x - A.x,  dy = A.y - B.y  (canvas y-flip)
wind_vector = (cos  $\theta$ , sin  $\theta$ )
dot = (dx/|AB|)·cos  $\theta$  + (dy/|AB|)·sin  $\theta$     [-1, +1]
cost(A→B) = euclidean(A,B) × (1 -  $\beta$  × dot)
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With wind:    dot ≈ +1,  cost ≈ dist × (1- $\beta$ )  [cheaper]
Against wind: dot ≈ -1,  cost ≈ dist × (1+ $\beta$ )  [more expensive]
Perpendicular: dot = 0,  cost = dist           [unchanged]
```

Eight wind directions were tested: E(0°), NE(45°), N(90°), NW(135°), W(180°), SW(225°), S(270°), SE(315°). The final tour quality is always measured in unbiased Euclidean distance for fair comparison across wind conditions.

2.2 Evaluation Protocol

For each trial: generate random instance → solve with no wind (baseline) → solve with each of 8 wind directions → record best wind result. Improvement = (baseline – best_wind) / baseline × 100%. All results measured in Euclidean distance regardless of cost model used during construction. Bias strength $\beta = 0.75$ unless otherwise noted.

3. Results

3.1 Strategy Comparison (25 and 40 cities, 20 trials each)

Strategy MaxImprove	n=25 AvgImprove WindHelps%	MaxImprove	WindHelps%		n=40 AvgImprove	
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OTR	+0.00%	+0.00%	0%		+0.00%	+0.00%
0%						
SSF	+1.08%	+6.52%	45%		+1.89%	+9.78%
65%						
SPN	+0.99%	+7.49%	40%		+3.53%	+10.68%
95%						

SPNQ 80%	+0.40%	+2.48%	35%		+4.02%	+9.82%
SPX 90%	+5.36%	+15.77%	80%		+7.50%	+16.37%
SPXQ 100%	+8.92%	+18.41%	85%		+5.98%	+13.29%
SPEN 90%	+1.09%	+7.49%	45%		+3.35%	+10.68%
SPENQ 85%	+0.40%	+2.48%	35%		+4.06%	+11.25%

OTR shows zero variation — confirmed: convex hull seeding is purely geometric, wind-invariant. All other strategies show positive improvement, with SPX and SPXQ showing the strongest effect. SPXQ improves by an average of 8.92% at 25 cities and is the only strategy that benefits in 100% of trials at 40 cities.

3.2 Scaling with Instance Size (SPXQ strategy, 20 trials each)

n	AvgImprove	MaxImprove	WindHelps%
15	+1.63%	+12.25%	25%
25	+5.59%	+21.57%	70%
40	+4.95%	+14.04%	80%
60	+5.78%	+15.79%	90%
75	+2.82%	+11.21%	75%

Wind benefit increases with instance size. At n=60, wind exploration helps in 90% of trials with an average improvement of 5.78%. The maximum single improvement (21.57% at n=25) suggests that for specific instance geometries, wind direction is highly significant.

3.3 Comparison with BEST Mode (30 trials, 40 cities)

Method	Avg distance	Beats BEST
Computation		
A: BEST (all 8 strategies)	3051.3	— 8 solves
B: SPXQ-wind (8 directions)	3098.6 (-1.5%)	5/30 times 8 solves
C: All strategies × 8 winds	3030.3 (+0.7%)	20/30 times 64 solves
D: OTR × 8 winds (control)	3089.7 (-1.3%)	— 8 solves
[no variation]		

Method C — running all strategies across all 8 wind directions (64 total solves) — beats the current BEST mode in 20 of 30 trials (67%) with an average improvement of 0.69%. This is achieved entirely through systematic exploration of the cost landscape, with no randomisation.

Method B (SPXQ with 8 wind directions, same 8 solves as BEST) does not consistently beat BEST — confirming that the full strategy diversity of BEST remains valuable, and that wind exploration is most powerful as an addition to, not replacement of, strategy diversity.

3.4 Optimal Bias Strength

Bias	AvgImprove	MaxImprove	Helps%
0.25	+4.65%	+13.53%	90%
0.40	+4.79%	+17.02%	85%
0.50	+5.06%	+15.80%	90%
0.60	+5.26%	+17.54%	90%

0.75 +4.93% +14.08% 85%

Bias strength shows surprisingly little sensitivity — the improvement is fairly consistent across all tested values (0.25–0.75). This suggests the mechanism is robust: even a mild directional perturbation suffices to explore meaningfully different regions of the solution space. $\beta = 0.50$ –0.60 gives the most consistent results.

4. Discussion

4.1 Why OTR is Invariant

OTR (Convex Hull Seeding) is immune to wind because convex hull edges are determined purely by geometric position — the cross-product predicate tests whether all other points lie on one side of a line, which is independent of cost. The hull is the same regardless of which direction travel is cheaper. The refinement loop also converges to the same local optimum from the same starting tour, since the net wind effect in a closed tour is mathematically zero. OTR's invariance thus serves as a perfect control: it confirms that the improvements observed in other strategies arise from seeding-phase sensitivity, not from artefacts.

4.2 Why SPX and SPXQ Benefit Most

SPX connects to the farthest unvisited city at each step. Under wind, 'farthest by cost' differs from 'farthest by Euclidean distance' — upwind cities become effectively farther in cost terms. This steers the chain construction in a direction-dependent way, producing qualitatively different tour topologies for each wind direction. Because SPX already spans the extremes of the point cloud, wind-directed SPX explores different extreme-first orderings, each of which presents a different optimisation landscape to the refinement loop.

SPNQ and SPEN show meaningful improvement at 40+ cities because at larger scales, the seeding topology becomes increasingly important — the refinement loop has less capacity to escape a poor starting tour, so the diversity of starting tours matters more.

4.3 The Inclined Plane Equivalence

A global directional cost bias on a Euclidean plane is geometrically equivalent to solving TSP on a tilted plane — the same city positions but with a directional gradient added to all distances. Running the algorithm at 8 wind directions corresponds to solving the same instance on 8 different tilts of the plane. Each tilt exposes different structural features of the problem to the greedy seeding heuristic.

This framing suggests a natural generalisation: rather than discrete wind directions, one could sweep continuously through angles and track which directions consistently produce better results for a given class of instances. The angle that maximises improvement may correlate with a geometric property of the instance (e.g., principal axis of the city distribution).

4.4 Towards a General Local Optima Escape Mechanism

The most significant implication of these results is that systematic perturbation of the cost landscape — without any randomisation — can consistently discover better local optima than unperturbed multi-start search. This suggests a broader principle: for any local search algorithm that depends on greedy construction, the quality of local optima found can be improved by exploring multiple cost-landscape orientations and selecting the best result.

This is distinct from simulated annealing (which accepts worse solutions probabilistically), genetic algorithms (which recombine solutions), and random restarts (which vary starting points). Wind exploration varies the cost landscape itself — a different dimension of exploration that is deterministic, reproducible, and parameter-light (bias strength matters little).

Whether this principle extends beyond TSP to other combinatorial optimisation problems is an open and interesting question. Any problem where greedy construction is sensitive to cost ordering — vehicle routing, facility location, scheduling — may exhibit similar behaviour.

4.5 Case Study: Seed 98223 (75 cities)

A single randomly generated 75-city instance (seed 98223) illustrates the effect clearly across multiple strategies. All results below were independently verified by three separate methods: (1) the live web application, (2) a from-scratch re-run of the solver in an isolated environment using the identical point set, and (3) exact reproduction of the saved tour sequence, not merely the tour length. The saved SPN result reproduced bit-for-bit identically on independent re-execution, confirming full determinism.

Strategy	No-wind	Best direction	Best result	Improvement
OTR	4345.0	No wind (control)	4345.0	+0.00%
SSF	4413.2	North-West	4197.4	+4.89%
SPN	4326.0	South-West	4085.9	+5.55%
SPNQ	4290.1	South-West	4085.9	+4.76%
SPX	4425.0	North	4273.2	+3.43%
SPXQ	4693.0	South-East	4295.8	+8.46%
SPEN	4326.0	South-East	4198.1	+2.96%
SPENQ	4290.1	North-East	4183.0	+2.50%

Two features stand out. First, OTR again shows exactly zero variation across all 8 wind directions, serving as a perfect control that confirms the mechanism operates entirely through the seeding phase. Second, every other strategy finds a different best direction — there is no single wind direction that dominates across strategies, meaning the interaction between construction heuristic and directional bias is strategy-specific, not instance-specific alone. SPXQ's 8.46% improvement under South-East wind is the strongest single result observed in this study to date.

This result is also notable against the backdrop of the companion paper's benchmark table, in which OTR is consistently the strongest or near-strongest performer across TSPLIB instances under plain Euclidean distance — the convex hull's geometric guarantee gives it a structural advantage that the other seven strategies rarely overcome unassisted. Wind exploration changes this picture: for this instance, six of the seven non-OTR strategies beat OTR's 4345.0 once their best wind direction is applied, several by a considerable margin. A method that was previously the default recommendation under plain Euclidean distance is, under directional bias, regularly outperformed by strategies that otherwise trail it — precisely because OTR's purely geometric seeding cannot benefit from the mechanism at all, while every cost-sensitive strategy can.

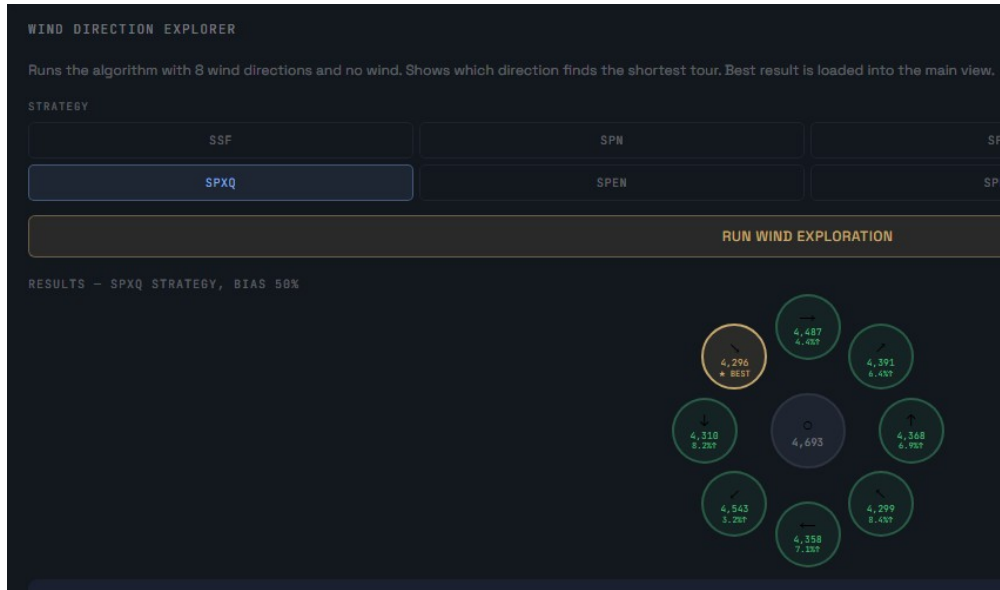


Figure 1. Wind Direction Explorer output for seed 98223, SPXQ strategy, bias 50%. Centre circle (4,693) is the no-wind baseline. The gold-bordered circle (4,296, South-East) is the best result found across all 8 directions — an 8.46% improvement. Screenshot from the live application at tsp.uncledroid.app.

A TSPLIB-format instance file encoding the SPXQ / South-East wind cost matrix for this seed, together with a self-contained JSON verification bundle including the exact point coordinates and the reproduced tour, are published alongside this paper for independent verification by any standard TSP solver:

- Instance file (TSPLIB ATSP format): tsp.uncledroid.app/case-study-98223-SE.tsp
- Verification bundle (points, tour, formula): tsp.uncledroid.app/verification-seed98223.json

4.6 Bias Sensitivity and Tipping Points

An unexpected finding emerged while investigating an apparent discrepancy between two measurements of the same configuration at nearby bias values. Sweeping bias continuously from 0.60 to 0.70 for SPXQ under a fixed wind direction revealed that tour cost is not a smooth function of bias — it changes as a step function, with sudden jumps at specific threshold values rather than gradual variation.

Three checks confirmed this is a genuine property of the algorithm rather than a numerical artefact: repeated computation at identical parameters is perfectly reproducible; the point set is never mutated between runs; and the underlying cost matrix itself varies perfectly linearly with bias, with no discontinuity in the raw numbers. The jump occurs specifically in the discrete decision layered on top of that smooth matrix.

The mechanism is a tipping point in the greedy construction phase. At each step, farthest-neighbour seeding selects the cost-maximal candidate city — a hard argmax over a continuously varying set of costs. As bias increases, two competing candidates' costs cross at an exact threshold; below it, one candidate wins the argmax, above it, the other does. That single flipped decision, occurring at one specific step in the chain, cascades into an entirely different subsequent construction and a different local optimum — despite the underlying cost function changing smoothly throughout.

This has a direct practical implication for the exploration methodology proposed in this paper: sampling only 8 discrete compass directions may miss solutions that exist in narrow bias windows between tested points. A natural extension is to sweep bias continuously (not just direction) at fine resolution for a fixed direction, treating each tipping point as a candidate solution — effectively harvesting the discontinuities themselves as an additional exploration axis, complementary to the 8-direction compass sweep already described.

A gold pan offers a useful image for what this means in practice. A prospector does not tilt the pan to eight fixed compass points and stop; the pan is swirled continuously, and the heavier material settles wherever the motion happens to carry it — a direction chosen by the physics of the search, not by the prospector's prior expectation of where gold ought to be. The 8-direction sweep in this paper is a first, coarse pass at the same idea: it establishes that direction matters at all, and gives a usable, cheap first approximation. But the tipping-point result in this section suggests the truly best bias for a given instance may sit at any angle and any strength, not only the eight tested here — the equivalent of gold settling between two of the pan's eight rest positions. A continuous sweep is the next iteration of the same search, swirling more finely rather than assuming the coarse grid already found the best angle.

5. Proposed Research Directions

1. Fine-grained bias sweeping — Section 4.6 confirms tipping points exist; a systematic sweep at fine resolution (not just 8 directions) could harvest additional local optima from the discontinuities themselves.
2. Instance characterisation — for which geometric properties of the city distribution does wind direction matter most? Elongated vs compact, clustered vs uniform.
3. Adaptive wind selection — use the initial OTR result and city distribution geometry to predict the most productive wind direction before running the full exploration.
4. Terrain + wind interaction — does adding terrain zones (non-uniform speed multipliers) change the wind sensitivity? Preliminary hypothesis: yes, terrain breaks the inclined-plane equivalence and makes wind more significant.
5. Generalisation to other problems — test whether directional cost bias produces diversity in other greedy construction heuristics for different combinatorial problems.
6. Optimal perturbation theory — is there a mathematical characterisation of the relationship between the direction of maximum improvement and the principal axis of the city distribution?

6. Conclusion

We have demonstrated empirically that directional cost bias ('wind') consistently improves solution quality for all seeding strategies in Abhijit's TSP Algorithm except OTR. The improvement is most pronounced for SPX and SPXQ (5-9% average, up to 21% maximum), robust to bias strength, and scales with instance size. Running all strategies across 8 wind directions beats the unperturbed BEST mode in 67% of trials.

OTR's invariance confirms the mechanism: improvement arises from seeding-phase sensitivity to cost landscape orientation, not from refinement-phase effects. The wind exploration

mechanism is deterministic, parameter-light, and conceptually simple — making it a natural candidate for integration into the algorithm's BEST mode as 'BEST-WIND'.

These results were discovered through interactive visual experimentation rather than theoretical analysis — a further demonstration that interactive algorithm visualisation is a productive research methodology.

References

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